

The outlier pixel and sub trees that form segments of image

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Abstract. Segmentation is a very important step in computer vision. Many segmentation methods are found to get the best results. Every method found always has advantages and disadvantages. Minimum spanning tree is one method used to segment images. Some researchers use thresholds as linear combinations of averages and standard deviations for this method. This study aims to see the effect of outliers on the number of segments formed. This study uses a sample of a 5x5 image with an outlier. Suppose that the distribution of side weights (or the difference in intensity of adjacent pixels) is a normal distribution except for one outlier, and the threshold value is equal to the average. If the weight of the outlier data side is equal to 14 to 16, there are 6 segments formed, for weights equal to 17 to 37, there are 4 segments formed, and for weights equal to 38 or more, there are 2 segments formed. The threshold value (corresponding average) changes significantly by changing the value of one outlier data. This will cause changes in the number of segments in the image that are not good in the segmentation method. So the threshold value should not correspond to statistical values.

1. Introduction

Segmentation is a very important step in computer vision. Almost every image processing method uses segmentation as part of the steps. For example for the step method of regional growth [1, 2], segmentation is used in the initial step of the method. The bandwidth value affects the area of each segment of the image. The area of each segment becomes large, if the bandwidth value becomes smaller. The segmentation method wants that a collection of several small segments corresponding to a bandwidth value will form a large segment corresponding to a smaller bandwidth value. But the mean shift method doesn't work like that [3].

The method of the minimum spanning tree [4] proposed by Peter is the answer to improving the segmentation method. In this method, the image is seen as a grid graph as in Figure 1. Pixels in the image are points on the grid graph. Each point is connected with four points around it. The side weight that connects two neighboring points is the difference in the intensity value between these two points. In this paper, the intensity used is the intensity of gray colors, ie values from 0 to 255.

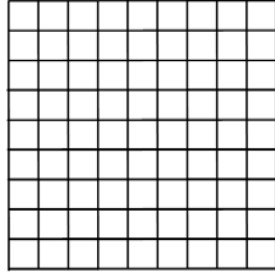


Figure 1. Grid Graph of the Image

Suppose the distribution of the pixel intensity of an image is normally distributed, except for one pixel as outliers. The detection of outliers has been a concern for statisticians and other scientists for more than a century [5]. Peter [4] uses a linear combination of averages and standard deviations from side-by-weight data as threshold values. Outliers data will be placed in the corner of the image. The intensity values will be investigated for various values. The purpose of this research is to find out the effect of outlier data values on the number of segments formed in the image.

This paper is arranged in several sections. The first section begins by discussing the problems to be solved in this study. The second section talks about the minimum spanning tree and the uniqueness of segmentation. The results and discussion are written in the third section. The last section contains conclusions from this paper.

2. The Graph and Segmentation

Image is seen as a graph grid, where pixels of an image are points in a graph, and each point is connected with four points around which the weight is the same as the difference in intensity between the two points connected. The symbol of the grid graph is written the same as the definition of graph [6], namely: $G = (V, E)$, where $V = V(G)$ is a set of points G , and $E = E(G)$ is a set of side G . If e is a member of set E , symbol $w(e)$ means weight for side e . The number of sides connected to a point v is called the degree of v , and is written with $deg(v)$. In accordance with Figure 1, the vertex has a degree 2, the other outside points have 3 degrees, while the points inside have degrees 4. So this grid graph has only three trees that are not homorphic for 5-points [7].

The Tree is a connected graph that does not have a cycle. All tree properties [8, 9] that will support this paper will be discussed first. The first property, let T graph has n points and m sides. T is a tree if and only if T is connected and $n = m + 1$. Further properties, Graph T is a tree if and only if every two different points are connected by exactly one path. This says that if two neighboring points are connected then a cycle will be formed.

The tree spanning a graph is called the spanning tree of the graph. The spanning tree of a graph is not unique. The number of spanning trees of graph G with n points is $\kappa(G) = \det(J + PP^T) / n^2$, where J is a square matrix in which all elements are one and P is a matrix of connectedness. In getting the minimum spanning tree, the Kruskal algorithm can be used properly. This algorithm works by steps entering side by side with the smallest weight and the addition of these sides does not form a circle. Because spanning tree is not unique, the minimum spanning tree may also not be unique. According to Mayr [10], every minimum spanning tree can be generated using the Kruskal Algorithm.

Suppose that S and T are two different minimum spanning trees. The following theorem mentions the weight that distinguishes S and T and how they are traced in the graph if that side is added to the graph [11].

Theorem 1. *Let $G(V, E)$ a weight graph and S, T is a minimum spanning tree of G . If $e \in E(S) \setminus E(T)$, then $T + e$ contains exactly one circle C such that C contains edge $e' \in E(T) \setminus E(S)$ where $w(e) = w(e')$, $E(C - e) \subseteq E(T)$, $E(C - e') \subseteq E(S)$, and for each $e_i \in C$ applies $w(e_i) \leq w(e) = w(e')$.*

The following statement [12] is the theorem used to show the uniqueness of the segmentation of images that use a minimum spanning trees. This theorem says that the selection of the minimum spanning tree to segment images will not affect the segments produced in the image.

Theorem 2. *Let $\alpha \in \mathbb{R}$, and S, T be the minimum spanningTree of graph G . Suppose that all of the weights are greater than or equal to α removed from the trees S , and T . Then the components connect S_1, S_2, \dots, S_p from trees S and T_1, T_2, \dots, T_q of the T tree will be formed. Then $p = q$ and if $V(S_i) \cap V(T_j) \neq \emptyset$ then $V(S_i) = V(T_j)$.*

3. The Outlier Pixel and Segmentation

Research in computer vision often cuts images into smaller ones, so that data is analyzed more easily. To simplify the work, the object discussed in this study is a 5x5 grid graph. So that the graph consists of 25 points and 40 sides. Suppose the weight of the sides is normally distributed. This distribution was chosen because most of the data in the world is normally distributed. So that the data used in this study are as found in Table 1.

Table 1. Data Side Weight that is Normal Distribution

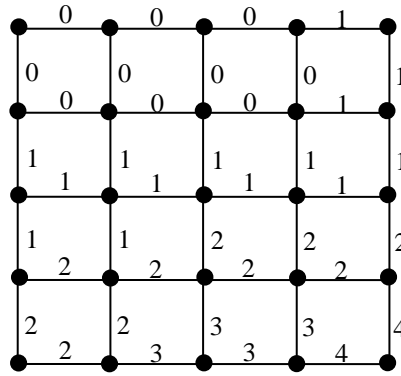
Side Weight	Frequency
-4	1
-3	2
-2	5
-1	7
0	10
1	7
2	5
3	2
4	1

The method of minimum spanning tree in segmenting uses weights with positive values. So that the data in Table 1, which has negative side weights, are changed to positive side weights such as the data in Table 2. There are 10 data with side weights equal to 0, 14 data with side weights equal to 1, 10 data with side weights equal to 2, 4 data with side weights equal to 3, and 2 data with side weights equal to 4. Because outliers are placed at the vertex, the data on the side weights of 4 in Table 2 will be changed the larger.

Table 2. Data with Positive Side Weight

Side Weight	Frequency
0	10
1	14
2	10
3	4
4	2
Total	40

The maximum number of segments will be obtained if we put the sides with the smallest weight to form the smallest circles. According to Theorem 1, one of the sides of this square must be removed to form a tree. To get the most number of segments, we will place the sides with the smallest weight starting from the upper left side of the square to the bottom right side of the square for the largest side weight as in Figure 2.

**Figure 2.** Graph forming maximum number of segments

The average and standard deviation of the graph side weight data in Figure 2 respectively are 1.35 and 1.09. If the average is chosen as a threshold value, then all sides with weights greater than 1.35 will be removed from the minimum spanning tree. Graphs with side weights less than 1.35 will form one sub-tree or one segment. While each other point will be seen as one segment. By calculating the points in Figure 2, the number of segments formed is equal to 9 segments. In the same way in case the threshold value is equal to the average minus the standard deviation (0.26), the number of segments formed is equal to 18 segments. Likewise, in the case of a threshold value equal to the average plus standard deviation (2.44), the number of segments formed is equal to 4 segments.

Suppose that the point in the lower right corner is the outlier point, namely: the two-sided weight connected to the outlier point is set to a large value. We will examine how it affects the number of segments of the image. For threshold values equal to the average minus standard deviation, the maximum number of segments is equal to 18 for side weights to outliers starting from 4 to 6, and the number of segments equals 25 for side weights equal to 7 or more. For the threshold value equal to the average, the maximum number of segments is equal to 9 for side weights to outliers starting from 4 to 16, and the number of segments equals 4 for side weights from 17 to 37, and the number of segments equal to 2 for side weights equal to 38 or more. For the threshold value equal to the average plus the standard

deviation, the maximum number of segments is equal to 4 for side weights to outliers starting from 4 to 6, and the number of segments is equal to 2 for side weights of 7 or more.

The threshold value (corresponding average) changes significantly by changing the value of one outlier data. This will cause changes in the number of segments in the image that are not good in the segmentation method. So the threshold value should not correspond to statistical values.

4. Conclusions

Suppose that the point in the lower right corner is the outlier point, namely: the two-sided weight connected to the outlier point is set to a large value. For threshold values equal to the average minus standard deviation, the maximum number of segments is equal to 18 for side weights to outliers starting from 4 to 6, and the number of segments equals 25 for side weights equal to 7 or more. For the threshold value equal to the average, the maximum number of segments is equal to 9 for side weights to outliers starting from 4 to 16, and the number of segments equals 4 for side weights from 17 to 37, and the number of segments equal to 2 for side weights equal to 38 or more. For the threshold value equal to the average plus the standard deviation, the maximum number of segments is equal to 4 for side weights to outliers starting from 4 to 6, and the number of segments is equal to 2 for side weights of 7 or more.

The threshold value (corresponding average) changes significantly by changing the value of one outlier data. This will cause changes in the number of segments in the image that are not good in the segmentation method. So the threshold value should not correspond to statistical values.

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